Coherent Change Detection for Multi-Polarization SAR

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Abstract

This paper presents a solution to the coherent change detection (CCD) problem using multi-polarization synthetic aperture radar (SAR) imagery. The multi-polarization SAR imagery (i.e., the day-1 reference and day-2 test images) are modeled as jointly correlated complex Gaussian vectors with unknown correlation, \( \rho = \gamma e^{i\phi} \). Maximum likelihood estimates of the unknown phase and coherence parameters \((\phi, \gamma)\) are derived. Accuracy of the MLE estimates is evaluated; and the benefit of using multi-polarization data versus single-polarization data is quantified.

Introduction

We investigate the use of fully polarimetric synthetic aperture radar (SAR) imagery in a coherent change detection application. The SAR is assumed to measure (for each pixel in the SAR image) three complex polarization returns: HH, HV, and VV; these three returns are elements of the three-dimensional complex vector, \( X \), which we model as a complex Gaussian vector of dimension “p=3”:

\[
X = \begin{pmatrix} HH \\ HV \\ VV \end{pmatrix}
\] (1)

The corresponding probability density (PDF) of vector, \( X \), is:

\[
f(X) = \frac{\exp\left(-\frac{X^\dagger C^{-1}X}{\pi^p |C|}\right)}{\pi^p |C|}
= \frac{\exp\left(-\text{tr}\left[C^{-1}XX^\dagger\right]\right)}{\pi^p |C|}
\] (2)

\( |C| \) is the determinant of the positive definite covariance matrix \( C = E[XX^\dagger] \) and \( X^\dagger \) is the complex conjugate transpose of the vector \( X \).

In this change detection application we assume there are two data sets: (1) the reference data \((X_1, X_2, \ldots, X_N)\), and (2) the mission data \((Y_1, Y_2, \ldots, Y_N)\). These data sets correspond to a reference image that was gathered on a previous day (day-1) and a more recently gathered (day-2) mission image. These two SAR images are assumed to be accurately pixel aligned; therefore we model the data by an augmented measurement vector, \( Z \):

\[
Z = \begin{pmatrix} X \\ Y \end{pmatrix}
\text{ where}
X = \begin{pmatrix} HH_x \\ HV_x \\ WV_x \end{pmatrix}
\text{ and } Y = \begin{pmatrix} HH_y \\ HV_y \\ WV_y \end{pmatrix}
\] (3)
Thus, the dimension of vector $Z$ is “2p”. Under a jointly Gaussian assumption, the PDF of measurement vector $Z$ is:

$$f(Z) = \frac{\exp\left(-Z^T Q^{-1} Z\right)}{\pi^{2p} |Q|} = \frac{\exp\left(-\text{tr}\left[Q^{-1} ZZ^T\right]\right)}{\pi^{2p} |Q|}$$

(4)

$|Q|$ is the determinant of covariance matrix:

$$Q = E[ZZ^T] = \begin{pmatrix} C & \rho C \\ \rho^* C & C \end{pmatrix}$$

(5)

As implied by the covariance matrix given in Equation 5, the data in image-1 and image-2 have the same covariance structure denoted by the common covariance matrix, $C$; the correlation parameter "\( \rho \)" denotes the correlation between the complex elements of the day-1 and day-2 measurements. We have observed in real SAR measurement data, typical correlation values of \( |\rho| \approx 0.9 \).

In coherent change detection, the metric used to decide whether or not a significant change has occurred between the day-1 and day-2 images is the value of the coherence parameter, \( \gamma = |\rho| \). When \( \gamma \to 1 \), the day-1 and day-2 images are very similar, implying no change has occurred; when \( \gamma \to 0 \), the day-1 and day-2 images are very dissimilar, implying a significant change has occurred. Therefore, our goal in the analysis that follows is to find the best (MLE estimate) of the coherence parameter, \( \gamma \), from the observed day-1 and day-2 SAR images.

As we shall see in the following section, in order to obtain this coherence estimate we will also obtain the MLE estimate of the phase parameter, \( \phi \), the relative phase between the day-1 and day-2 images.

### MLE estimates of the coherence and phase parameters

To obtain the maximum likelihood estimate of the coherence parameter, \( \gamma_{\text{MLE}} \), we write the conditional PDF (likelihood function) of our data measurements (conditioned on the unknown parameters \( \gamma, \phi \)) as follows:

$$f(Z_1, Z_2, ..., Z_N | \gamma, \phi) = \frac{\exp\left(-\sum_{k=1}^{N} Z_k^* Q^{-1} Z_k\right)}{\pi^{2NP} |Q|^N}$$

$$= \frac{\exp\left(-\text{tr}\left[Q^{-1} \sum_{k=1}^{N} Z_k Z_k^T\right]\right)}{\pi^{2NP} |Q|^N}$$

The covariance matrix inverse is given by the formula:

$$Q^{-1} = \begin{pmatrix} C^{-1} & -\gamma e^{i\phi} C^{-1} \\ -\gamma e^{-i\phi} C^{-1} & C^{-1} \end{pmatrix}$$

(7)

The determinant of the covariance matrix is given by the formula:

$$|Q| = (1 - \gamma^2)^p |C|^2$$

(8)

Evaluating the log-likelihood function gives:
\[
\ln f(Z_1, Z_2, \ldots, Z_N | \gamma, \phi) = -2Np \ln \pi - Np \ln(1 - \gamma^2) - 2N \ln |C|
\]
\[
-\frac{1}{(1 - \gamma^2)} trC^{-1} \left( \sum_{k=1}^{N} X_k X_k^\dag + \sum_{k=1}^{N} Y_k Y_k^\dag \right) + \frac{1}{(1 - \gamma^2)} trC^{-1} \left( \gamma e^{i\phi} \sum_{k=1}^{N} X_k X_k^\dag + \gamma e^{-i\phi} \sum_{k=1}^{N} X_k Y_k^\dag \right)
\]

To determine the MLE estimate of unknown parameter, \( \gamma \), we take the derivative
\[
\frac{\partial}{\partial \gamma} \ln f(Z_1, Z_2, \ldots, Z_N | \gamma, \phi) = 0 \quad (10)
\]

Taking the partial derivative and simplifying, we obtain the following expression (11):
\[
0 = \gamma (1 - \gamma^2) - \frac{2\gamma}{2Np} trC^{-1} \left( \sum_{k=1}^{N} X_k X_k^\dag + \sum_{k=1}^{N} Y_k Y_k^\dag \right) + \frac{(1 + \gamma^2)}{2Np} trC^{-1} \left( e^{i\phi} \sum_{k=1}^{N} Y_k X_k^\dag + e^{-i\phi} \sum_{k=1}^{N} X_k Y_k^\dag \right)
\]

Using the following reasonable approximations:
\[
\frac{1}{N} \sum_{k=1}^{N} X_k X_k^\dag \approx C \quad \text{and} \quad \frac{1}{N} \sum_{k=1}^{N} Y_k Y_k^\dag \approx C
\]
we obtain the result:
\[
trC^{-1} \left( \sum_{k=1}^{N} X_k X_k^\dag + \sum_{k=1}^{N} Y_k Y_k^\dag \right) \approx trC^{-1} (2NC) = 2Np
\]

Substituting the above approximation into Equation 11, we obtain:
\[
0 = \gamma (1 - \gamma^2) - 2\gamma + (1 + \gamma^2) \left[ \right] \quad (14)
\]

where the factor
\[
\frac{trC^{-1} \left( e^{i\phi} \sum_{k=1}^{N} Y_k X_k^\dag + e^{-i\phi} \sum_{k=1}^{N} X_k Y_k^\dag \right)}{trC^{-1} \left( \sum_{k=1}^{N} X_k X_k^\dag + \sum_{k=1}^{N} Y_k Y_k^\dag \right)} \approx \left[ \right]
\]

The MLE estimate of the coherence parameter, \( \gamma_{\text{MLE}} \), is a solution of the following cubic equation:
\[
0 = (1 + \gamma^2)(-\gamma + \left[ \right]) \quad (16)
\]

Finally, taking the unique real root as the solution gives expression (17):
\[
\gamma_{\text{MLE}} = \frac{trC^{-1} \left( e^{i\phi} \sum_{k=1}^{N} Y_k X_k^\dag + e^{-i\phi} \sum_{k=1}^{N} X_k Y_k^\dag \right)}{trC^{-1} \left( \sum_{k=1}^{N} X_k X_k^\dag + \sum_{k=1}^{N} Y_k Y_k^\dag \right)}
\]

Next we apply polarization “whitening” to the SAR measurement data as follows:
\[
\tilde{X}_k = C^{-\dag} X_k \quad \text{and} \quad \tilde{Y}_k = C^{-\dag} Y_k \quad (18)
\]

In the “whitened” measurement space, the MLE estimate of the coherence is given as follows:
\[ \gamma_{\text{MLE}} = \frac{\sum_{k=1}^{N} \left( \tilde{X}_k^\dagger \tilde{Y}_k e^{i\phi} + \tilde{Y}_k^\dagger \tilde{X}_k e^{-i\phi} \right)}{\sum_{k=1}^{N} \| \tilde{X}_k \|^2 + \sum_{k=1}^{N} \| \tilde{Y}_k \|^2} \]  

\( (19) \)

The above expression may be written in a more useful way by using the following:

\[ \sum_{k=1}^{N} \tilde{X}_k^\dagger \tilde{Y}_k = \sum_{k=1}^{N} \left( \cdot \right) = \left( \sum_{k=1}^{N} \tilde{X}_k^\dagger \tilde{Y}_k \right) \]  

\( (20) \)

Using the above result, the coherence estimate becomes:

\[ \gamma_{\text{MLE}} = \frac{2 \left| \sum_{k=1}^{N} \tilde{X}_k^\dagger \tilde{Y}_k \right| \cos \left( \phi - \left( \sum_{k=1}^{N} \tilde{X}_k^\dagger \tilde{Y}_k \right) \right)}{\sum_{k=1}^{N} \| \tilde{X}_k \|^2 + \sum_{k=1}^{N} \| \tilde{Y}_k \|^2} \]

\( (21) \)

The above expression is maximized with respect to the unknown phase parameter, \( \phi \), by taking as the MLE estimate:

\[ \phi_{\text{MLE}} = \left( \sum_{k=1}^{N} \tilde{X}_k^\dagger \tilde{Y}_k \right) \]

\( (22) \)

Finally, the following MLE estimate of the coherence parameter is obtained:

\[ \gamma_{\text{MLE}} = \frac{2 \left| \sum_{k=1}^{N} \tilde{X}_k^\dagger \tilde{Y}_k \right|}{\sum_{k=1}^{N} \| \tilde{X}_k \|^2 + \sum_{k=1}^{N} \| \tilde{Y}_k \|^2} \]

\( (23) \)

Figure 1 shows a block diagram of the signal processing steps performed in implementing the coherent change detection algorithm using multiple polarization SAR measurement data. As indicated by the block diagram of Figure 1, the reference and mission images (denoted as image-1 and image-2) are first processed using polarization whitening; this converts the measured complex polarization vectors from image-1, \( \tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N \), into an equivalent set of polarization vectors, \( \tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_N \) having orthonormal polarization elements (similar processing is performed on the image-2 polarization vectors). The whitening filter is denoted by the linear transformation \( C^{-\frac{1}{2}} \) (see Reference 1 for details). The final step of processing is performed on the whitened polarization vectors. The MLE estimates \( (\phi_{\text{MLE}}, \gamma_{\text{MLE}}) \) are obtained using the expressions given in the figure.

The multi-polarization coherent change detection solution given above in Equations 22-23 is valid for either single-polarization \( (p=1) \), dual-polarization \( (p=2) \), or full polarization measurements \( (p=3) \). In the next section of the paper we present a preliminary performance comparison of single-polarization versus full polarization implementations.
Simulated coherent change detection results

This section of the paper presents simulated coherent change detection performance results using computer generated multi-polarization data. The performance results presented here were obtained using a Monte Carlo simulation of the MLE estimators derived in the previous section of this paper (Equations 22 and 23). In the simulation we used a 5 by 5 “box” of pixels (N=25) and we simulated two sets of 25 independent, 3-dimensional complex Gaussian vectors representing the day-1, day-2 data vectors. We simulated day-1 and day-2 data vectors having true correlation parameter values $\phi = 60$ degrees and $\gamma = 0.8$. These data sets were obtained using a Matlab code that generated 6-dimensional complex Gaussian vectors having the following polarization covariance matrix:

\[
Q = \begin{bmatrix}
1 & 0 & 0.5 & 0.8e^{-i\phi} & 1 & 0 \\
0 & 0.2 & 0 & 0 & 0 & 0.5 \\
0.5 & 0 & 1 & 0 & 0 & 0 \\
0.8e^{-i\phi} & 0 & 0.2 & 0 & 0 & 0.5 \\
0.5 & 0 & 1 & 0.5 & 0.5 & 0 \end{bmatrix}
\]

The estimated phase and coherence parameters obtained from our Monte Carlo simulations are shown in Figures 2 and 3; the histograms show the accuracy of the MLE algorithms in estimating the phase and coherence parameters from the day-1 and day-2 data vectors for true parameter values of $\phi = 60$ deg and $\gamma = 0.8$, respectively. Using full polarization data, the standard deviations of the single-polarization estimation errors were found to be reduced by a factor of $\sqrt{3}$. 

Figure 1: Block diagram showing the signal processing steps performed in coherent change detection using multiple polarization SAR measurement data.
Figure 2: Histogram of the MLE phase parameter estimates using N=25 pixels. True phase angle $\phi = 60$ degrees; std. dev. of single-POL phase estimates is 6.1 deg; std. dev. of three-POL phase estimates is 3.7 deg. The histogram was obtained from 1000 independent Monte Carlo trials.

Figure 3: Histogram of the MLE coherence parameter estimates using N=25 pixels. True coherence = 0.8; std. dev. of single-POL coherence estimates is 0.055; std. dev. of three-POL coherence estimates is 0.033. The histogram was obtained from 1000 independent Monte Carlo trials.

Summary

This paper investigated the use of multi-polarization synthetic aperture radar imagery in a coherent change detection application. The multi-polarization reference and test images were modeled as jointly correlated Gaussian data with unknown complex correlation ($\rho = \gamma e^{-i\phi}$). Maximum likelihood estimates of the unknown parameters ($\phi, \gamma$) were derived and the accuracy of these MLE estimates was determined using a Monte Carlo simulation. Parameter estimation errors were compared using single-polarization (HH) data versus multi-polarization (HH, HV, VV) data. The standard deviation of the errors of the MLE estimates using single-polarization data was reduced by approximately $\sqrt{3}$ when multi-polarization (HH, HV, VV) data were used.

References

